CALCULATION OF THE EFFECTIVE THERMAL DIFFUSIVITY OF HETEROGENEOUS LAYERED MATERIAL

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Recommendations and limiting conditions were worked out for calculating the effective thermal diffusivity of layered composite materials.

Calculations of nonsteady-state temperature fields in heterogeneous materials entail considerable mathematical difficulties both in the case of analytical approach [1-3] and when numerical methods are used. These methods serve for examining discrete regions of the components with the corresponding thermophysical properties and boundary conditions according to the structural arrangement of the material, and the obtained solutions of the differential equation of thermal conductivity make it possible to determine the temperature at any point of the bulk of the composite material at any instant.

In many practically important problems there is no need of such complete information on the temperature fields in the bulk of the material, and it suffices to have information on the time-dependent change of temperature only in some section of the material or on the boundary of the body. For such problems it is expedient to determine the conditions under which the effect of each discrete region of the components on the inhomogeneity of the temperature field is negligibly small, and the aggregate of their effect can be taken into account by introducing some effective properties characterizing the heat transfer within the bulk of the composite material.

Such an approach has found widespread application in the theory of generalized conductivity [4] for the calculation of the effective thermal conductivity of heterogeneous materials. The real chaotic structure of the material is reduced to an adequate ordered structure, and owing to having a further order (regular recurrence of the geometric and physical properties of the structure) it is always possible to distinguish an elementary cell. The aggregate of regularly arranged elementary cells makes it possible to restore the entire volume of the initial ordered model.

The validity of the final calculation formulas is practically determined by the validity of the transition from inhomogeneous real material to the quasihomogeneous model with the aid of the elementary cell. Then we analyze by various methods the course of the thermal flow through the elementary cell for the steady-state thermal regime, and a formula is suggested for determining the effective thermal conductivity. There are no concrete recommendations and restrictions for the dimensions or for the number of elementary cells, except the requirement that $L \gg l$ (L is the characteristic dimension of the specimen, l is the characteristic dimension of the elementary cell). The volume of the elementary cell of the heterogeneous material, by which the temperature field and the properties are averaged, has to be sufficiently large in comparison with the microscopic inhomogeneities of the material, and very small in relation to the macroscopic inhomogeneities of the temperature field. Only in this case can the results of the calculation of the conductivity of the elementary cell be applied to inhomogeneous real material.

Thermal diffusivity, determined as the ratio of thermal conductivity to volumetric specific heat of the material [5], is an important thermophysical characteristic whose use helps solve many problems of nonsteady-state thermal regime.

In the transition from inhomogeneous real material to the quasihomogeneous model, to which effective thermal diffusivity is applicable, analogously to effective thermal conductivity, attention must be given not only to the adequacy of the elementary cell but also to the eventual dimensions of the cell and of the real specimen.

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Kerrisk [6] recommends the criterion of homogeneity for matrix structures in connection with the experimental investigation of thermal diffusivity by the methods of the regular regime of the third kind and of impulsive heating. According to the criterion of homogeneity, the diameter of an inclusion has to be much smaller (between 100 and 1000 times) than the thickness of the specimen. It is characteristic that the criterion of homogeneity contains only the geometric dimensions, and that the difference in the properties of the components of the material is not taken into account. It is easy to realize that when the properties of the components differ only slightly, then this criterion is excessively stringent.

In the course of the investigations of [7, 8], the conditions of homogeneity for matrix structures in the form of ratios of the geometric characteristics of the structure and the physical properties of the components were experimentally discovered. The material was considered homogeneous when the thermal diffusivity, measured by the method of impulsive heating, coincided within the limits of the error with the thermal diffusivity found with the aid of the correlation equation

$$a_{ef} = \lambda_{ef}/(c\rho)_{ef}, \tag{1}$$

where a_{ef} is the effective thermal diffusivity; λ_{ef}, effective thermal conductivity; $(c\rho)_{ef}$, volumetric specific heat.

The effective thermal conductivity was determined according to [9], and specific heat by the rule of additivity. It was found that the investigated materials (ratio of the thermal diffusivities of the components 0.48—1137, of the thermal conductivities 9.5—2370) are homogeneous although the criteria type [6] were not met. The ratio of the diameter of the inclusion to the thickness of the specimen was between 0.1 and 0.25.

Dul'nev and Sigalov [10] presented the mean volumetric and rms errors of determining the dimensionless temperature occurring in the transition from inhomogeneous to quasihomogeneous materials. They examined layered materials with different arrangement of the layers in relation to the direction of the heat flow. The results of the numerical solution indicate palpable errors, especially when the material has a small number of layers.

In addition to the geometric characteristics of the structure and the physical properties of the components, the criterion of homogeneity also depends of course on the type of structure.

Henceforth we will examine two-component materials consisting of plates (layers) orientated perpendicularly to the direction of the thermal flow. Adjacent layers are situated in conditions of ideal thermal contact, and between them physicochemical transformations do not occur.

The selection of the layered structure of the material is not only due to the endeavor to simplify analysis but also to the closeness of real problems and materials to similar structures.

Schimmel et al. [11] present an expression for calculating effective thermal diffusivity of composite material with the above-mentioned structure consisting of N layers; the expression has the form

$$\frac{L^2}{a_{ef}} = \left[\sum_{i=1}^{N} \frac{l_i^2}{a_i} + 2\sum_{i=1}^{N} \frac{l_i}{a_i} \sum_{j=i+1}^{N} \frac{c_j\rho_j l_j}{c_i\rho_i}\right], \tag{2}$$

where i is the ordinal number of the layer; a, thermal diffusivity; ρ, density; c, specific heat per unit weight; l, thickness of the layer; L, overall thickness of the material in the direction of the heat flow.

Expression (2) was obtained by the approximate method of superposition for the case when one surface (the rear one) of layered composite material is heat insulated (adiabatic), and on the other surface (the front one) the heat flow can change arbitrarily in time. By using (2), we can determine a change of temperature in time only on the front and rear surfaces of the layered plate; such a limitation ensues from the simplifications adopted in the statement of the problem by Schimmel et al. [11].

Let us examine a two-component layered composite material (Fig. 1) consisting of n layers of each component. Then N = 2n, and if we adopt a specimen of unit length (L = 1), we have:

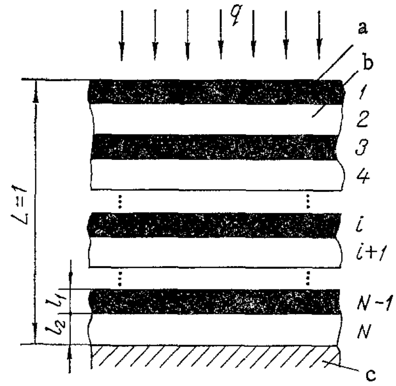

Fig. 1

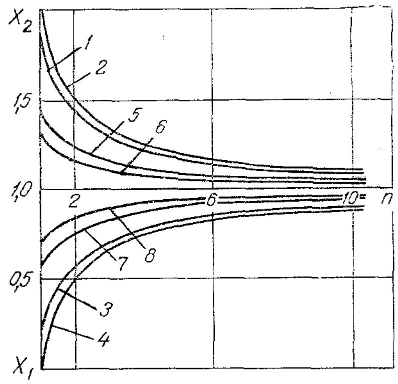

Fig. 2

Fig. 1. Diagram of a multilayered two-component material: a) component 1; b) component 2; c) adiabatic surface; q) direction of heat flow; numbers on the right: layer numbers.

Fig. 2. Dependence of the criterion of homogeneity on the number of layers and on the volumetric concentrations of the components: 1, 2, 3, 4) thermophysical properties of the components: $\lambda_1 = 418.6$ W/(m·°K), $a_1 = 3\cdot 10^{-5}$ m²/sec, $(c\rho)_1 = 1.395\cdot 10^7$ J/(m³·°K), $\lambda_2 = 0.042$ W/(m·°K), $a_2 = 3\cdot 10^{-7}$ m²/sec, $(c\rho)_2 = 1.40\cdot 10^6$ J/(m³·°K) [11]; 5, 6, 7, 8) thermophysical properties of the components: $\lambda_1 = 115$ W/(m·°K), $a_1 = 3.18\cdot 10^{-5}$ m²/sec, $(c\rho)_1 = 3.615\cdot 10^6$ J/(m³·°K), $\lambda_2 = 9.0$ W/(m·°K), $a_2 = 2.15\cdot 10^{-6}$ m²/sec, $(c\rho)_2 = 4.186\cdot 10^6$ J/(m³·°K); 1, 3, 5, 7) $m_2 = 0.1$; 2, 4, 6, 8) $m_2 = 0.5$.

$$l_1 = \frac{m_1}{n}; \quad l_2 = \frac{m_2}{n}. \tag{3}$$

We adopt $\lambda_1 > \lambda_2$, $a_1 > a_2$, $c_1\rho_1 \geqslant c_2\rho_2$, i.e., we consider component 1 nominally conducting, and component 2 to be insulating material.

Let the first layer consist of component 1, then the effective thermal diffusivity for the n-layered two-component material is expressed as

$$\frac{1}{a_{\text{ef}.1}} = n^2\left(\frac{l_1^2}{a_1} + \frac{l_2^2}{a_2}\right) + n(n+1)\frac{l_1}{a_1}\left(\frac{l_2c_2\rho_2}{c_1\rho_1}\right) + n(n-1)\frac{l_2}{a_2}\left(\frac{l_1c_1\rho_1}{c_2\rho_2}\right). \tag{4}$$

When the first layer is made of component 2, then

$$\frac{1}{a_{\text{ef}.2}} = n^2\left(\frac{l_1^2}{a_1} + \frac{l_2^2}{a_2}\right) + n(n-1)\frac{l_1}{a_1}\left(\frac{l_2c_2\rho_2}{c_1\rho_1}\right) + n(n+1)\frac{l_2}{a_2}\left(\frac{l_1c_1\rho_1}{c_2\rho_2}\right). \tag{5}$$

Expressions (4) and (5) were obtained for two-component material by the method of superposition explained in [11], and with the use of the method of mathematical induction for extension to the n-layered case.

It follows from (4) and (5) that the effective thermal diffusivity depends on the sequence in which the layers of the components are arranged. This deduction leads to the conclusion that the experimental data depend on the disposition of the specimens relative to the heating element in some methods of experimentally determining thermal diffusivity [7, 12].

It is known that effective thermal conductivity and volumetric specific heat, and according to (1) also effective thermal diffusivity, are invariant to the sequence in which the layers are arranged. It is easy to see that when the number of layers n increases, the results of calculation by (4) and (5) approach each other, and the dependence of the effective thermal diffusivity on the sequence of arrangement of the layers of the components gradually degenerates.

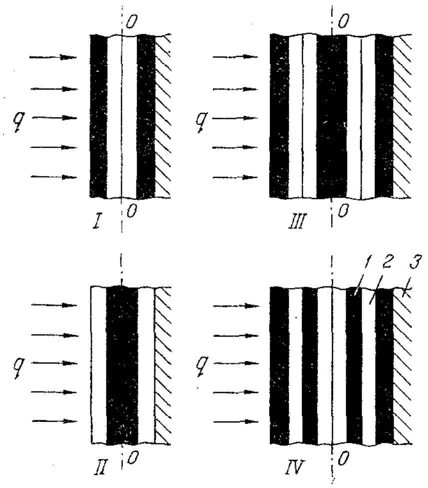

Fig. 3. Variants of the structure of layered composite material: 1) component 1; 2) component 2; 3) adiabatic surface; 0—0) central axis of the material; q) direction of heat flow; I, II) n = 2, N = 4; III, IV) 4 and 8, respectively.

According to the theory of generalized conductivity, the effective thermal conductivity of the given layered material is equal to

$$\lambda_{\text{ef}} = L \Big/ \left[n \left(\frac{l_1}{\lambda_1} + \frac{l_2}{\lambda_2} \right) \right]. \tag{6}$$

Volumetric specific heat is expressed as

$$(c\rho)_{\text{ef}} = \frac{n}{L} (c_1\rho_1 l_1 + c_2\rho_2 l_2) \tag{7}$$

and effective thermal diffusivity according to (1) is

$$\frac{1}{a_{\text{ef}}} = \left[n^2 \left(\frac{l_1^2}{a_1} + \frac{l_2^2}{a_2} \right) + n^2 \frac{l_1}{a_1} \left(\frac{l_2 c_2 \rho_2}{c_1 \rho_1} \right) + n^2 \frac{l_2}{a_2} \left(\frac{l_1 c_1 \rho_1}{c_2 \rho_2} \right) \right]. \tag{8}$$

When n increases, the numerical results according to (4) and (5) approach those of (8).

Thus, we can propose a criterion of homogeneity for two-component material with simple alternation of layers arranged perpendicularly to the direction of the heat flow (Fig. 1) which requires coincidence (proximity) of the numerical values of thermal conductivity found by (4), (5) on the one hand, and (8) on the other hand. We point out that in this case the criterion of homogeneity of the material contains the geometric as well as the physical properties of the components. From (4), (5), and (8) the criteria of homogeneity X_1, X_2 are equal to:

$$X_1 = \frac{a_{\text{ef}}}{a_{\text{ef}\,1}} = \frac{n^2 \left(\frac{l_1^2}{a_1} + \frac{l_2^2}{a_2} \right) + n(n+1) \frac{l_1}{a_1} \left(\frac{l_2 c_2 \rho_2}{c_1 \rho_1} \right) + n(n-1) \frac{l_2}{a_2} \left(\frac{l_1 c_1 \rho_1}{c_2 \rho_2} \right)}{n^2 \left[\frac{l_1^2}{a_1} + \frac{l_2^2}{a_2} + l_1 l_2 \left(\frac{c_2 \rho_2}{a_1 c_1 \rho_1} + \frac{c_1 \rho_1}{a_2 c_2 \rho_2} \right) \right]},$$

$$X_2 = \frac{a_{\text{ef}}}{a_{\text{ef}\,2}} = \frac{n^2 \left(\frac{l_1^2}{a_1} + \frac{l_2^2}{a_2} \right) + n(n-1) \frac{l_1}{a_1} \left(\frac{l_2 c_2 \rho_2}{c_1 \rho_1} \right) + n(n+1) \frac{l_2}{a_2} \left(\frac{l_1 c_1 \rho_1}{c_2 \rho_2} \right)}{n^2 \left[\frac{l_1^2}{a_1} + \frac{l_2^2}{a_2} + l_1 l_2 \left(\frac{c_2 \rho_2}{a_1 c_1 \rho_1} + \frac{c_1 \rho_1}{a_2 c_2 \rho_2} \right) \right]} \tag{9}$$

or

$$X_1 = 1 - a_{\text{ef}} \frac{m_1 m_2}{n} \left(\frac{c_1 \rho_1}{a_2 c_2 \rho_2} - \frac{c_2 \rho_2}{a_1 c_1 \rho_1} \right),$$

$$X_2 = 1 + a_{\text{ef}} \frac{m_1 m_2}{n} \left(\frac{c_1 \rho_1}{a_2 c_2 \rho_2} - \frac{c_2 \rho_2}{a_1 c_1 \rho_1} \right). \tag{10}$$

Numerically, the values of the criterion of homogeneity lie within the limits $0 \leq X_1 \leq 1$; $1 \leq X_2 \leq 2$. The values of X_1, X_2 approach unity, which corresponds to the fulfillment of the requirement that the material be homogeneous when the number of layers n increases, with insignificant volumetric proportion of one component, and in case of the difference between the thermophysical characteristics of the components being small.

As an example, Fig. 2 presents the values of the criteria of homogeneity for two different materials. The dependence of X_1, X_2 on the volumetric proportions of the components is monotonic because with $m_1 = 1$ or $m_2 = 1$, which corresponds to homogeneous material, the criteria of homogeneity are equal to unity.

When there is a substantial difference in the thermophysical properties of the components and when the number of layers is constant, the criterion of homogeneity deviates most from unity in the region of volumetric concentrations $m_1 \approx m_2 \approx 0.5$ (Fig. 2, curves 2 and 4). Similar results were also obtained theoretically and experimentally for a structure with layers arranged parallel to the direction of the heat flow [10, 13], and theoretically for the matrix structure [14].

It should be pointed out that when there are considerable differences in the thermophysical properties of the components, or even when the number of layers n is very large, the values of X_1 and X_2 may substantially differ from unity. This again puts emphasis on the necessity of carefully selecting the elementary cell for calculating effective thermal diffusivity of heterogeneous materials by (1) and of the corresponding relationship to the structure and thickness of the test specimens in the experimental determination of thermal diffusivity [12]. Analogous problems also arise in the experimental investigation of thermal conductivity by nonsteady-state methods [15].

Schimmel at al. [11] used a numerical method to find the dependence of the change of temperature on time for a two-component four-layer material with different combinations of the sequence of the component layers. It is characteristic that the effective thermal diffusivity of the material is greater when the layer of component 1 (the conducting component) faces the side of thermal action on the specimen. We analyzed (2) in connection with a two-component material. The point of departure of the analysis was to seek such regularities of the sequence of component layers (the number of layers of both components is equal) that the effective thermal diffusivity calculated by (2) coincides with the results of calculation by (8). In that case the criterion of homogeneity as the ratio of the mentioned thermal diffusivities is equal to unity.

Figure 3 presents several variants of structures that are axisymmetric with respect to the axis 0—0. Using the criterion of symmetry (equality of the parameters of the i-th and N + 1 — i-th layers $a_1 = a_{N+1-i}$; $l_1 = l_{N+1-i}$, etc.), we transform the ratio (2) into

$$\frac{L^2}{a_{\text{ef}}} = 2\sum_{i=1}^{n} \frac{l_i^2}{a_i} + 2\sum_{i=1}^{n} \left\{ \frac{l_i}{a_i c_i \rho_i} \left[2 \sum_{j=1}^{n} (c_j \rho_j l_j - c_i \rho_i l_i) \right] \right\}. \tag{11}$$

For the two-component case we can obtain (8) from expression (11). Hence follows that when the condition of symmetry is met, the thermal diffusivities calculated by (2), (11), and (8) coincide. This assertion is correct for any arbitrary volumetric proportion of the components, for any number of layers n, and independently of the relationship between the thermophysical properties of the components.

It must be pointed out that expression (11) may also be used in the more general case, for calculating the effective thermal diffusivity of a multicomponent axisymmetric material.

In the calculation of the change of temperature using (8), Schimmel et al. [11] were in error; in reality the corresponding curve has to coincide with the curves for the two cited axisymmetric layered structures.

Thus, the selection of a representative elementary cell and the requirements that the specimens of material have to meet in the investigation of the effective thermal diffusivity have similar features. It should be emphasized that the presented recommendations are correct with a view to the adopted assumptions, i.e., in the calculation of temperature on the rear adiabatic surface of the layered material. Problems of this and similar type are characteristic of some types of thermal insulation, and also in connection with the determination of thermal diffusivity by methods of impulsive and monotonic heating.

It is interesting to note that even in the case of one type of structure (layered composite materials with layers arranged perpendicularly to the direction of the heat flow) fulfillment of the criterion of homogeneity depends substantially on the regularity of the sequence of the layers, but for some combinations (axisymmetric sequence of layers) it is fulfilled automatically.

NOTATION

m_1, m_2, volumetric concentrations of components 1 and 2, respectively; λ_1, λ_2, thermal conductivity of components 1 and 2, respectively; a_1, a_2, thermal diffusivity of components 1 and 2, respectively; $(c\rho)_1$, $(c\rho)_2$, volumetric specific heat of components 1 and 2, respectively; q, heat flow; l_1, l_2, thickness of the layer of components 1 and 2, respectively; L, total thickness of the specimen of the material; $a_{ef.1}$, effective thermal diffusivity of heterogeneous material when the first layer is made of component 1; $a_{ef.2}$, the same for component 2; n, number of layers of the given component; N, total number of layers of components; X_1, X_2, criterion of homogeneity of the material.

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